**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

To solve this problem, we can use the standard normal distribution since the time required for servicing transmissions is normally distributed with a known mean and standard deviation.

Let X be the time required for servicing transmissions. We are given that X follows a normal distribution with mean (μ) = 45 minutes and standard deviation (σ) = 8 minutes.

The service manager commits to having the car ready within 1 Hour from drop-off. And the service manager begins the work on the transmission of a customer’s car 10 minutes after the car is dropped off. Therefore, we are interested in finding the probability that X is greater than 50 minutes.

First, let’s calculate the z-score using the formula:

Z = (X – μ)/ *σ*

where:

- X is the time commitment (50 minutes),

- μ is the mean (45 minutes),

- *σ* is the standard deviation (8 minutes).

Z = (50 – 45)/8 = 5/8

Now, we need to find the probability that X > 50, which is equivalent to finding the probability that Z > 5/8.

Now, we can use the probability in the RStudio using the “pnorm” function as shown below.

probability <- 1 - pnorm(5/8)

print(probability)

“pnorm” function helps us find the cumulative probability up to that z-score, and finally subtract it from 1 to find the probability that X > 50.

After running the code, we get the output 0.2659855, which is approximately 0.2676.

So, the answer is B. 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

Mean = 38

SD = 6

Z score = (Value – Mean)/SD

Z score for 44 = (44 – 38) / 6 = 1

In terms of percentiles, a z-score of 1 means the data point is at the 84.13th percentile.

People above 44 ages = 100 – 84.13 = 15.87% (Around 63 people, 15.87% of 400)

Z score of 38 = (38 – 38) / 6 = 0

In terms of percentiles, a z-score of 0 means the data point is at the 50th percentile.

So, the number of people between 38 and 44 years old = 84.13 – 50 = 34.13% (Around 137, 34.13% of 400)

Hence, Statement A. “More employees at the processing center are older than 44 than between 38 and 44” is False.

Z score of 30 = (30 – 38) / 6 = -1.33

In terms of percentiles, a z-score of -1.33 means the data point is at the 9.18th percentile.

So, the number of people below age 30 = 36 (9% of 400)

Hence, Statement B. “A training program for employees under the age of 30 at the center

would be expected to attract about 36 employees “is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

The difference between 2 *X*1 and *X*1 + *X*2 is*N*(0, 6σ2)

Step-by-step explanation:

According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean, μ, and the variance, σ2 and written as *X* ~ *N*(μ, σ2).

Given, *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are two independent identically distributed variables.

From the properties of normal random variables, if *X* ~ *N*(μ1, σ12) and *Y* ~ *N*(μ2, σ22) are two independent identically distributed random variables then

* the sum of normal random variables is given by

X + Y ~ *N*(μ1 + μ2,, σ12 + σ22)

* and the difference of normal random variables is given by

X - Y ~ *N*(μ1 - μ2,, σ12 + σ22)

* When Z = aX, the product of X is given by

Z ~ *N*(aμ1, a2σ12)

* When Z = aX + bY, the linear combination of X and Y is given by

Z ~ *N*(aμ1 +bμ2, a2σ12 +b2σ22)

Given to find, 2X1

Thus, following the property of multiplication, we get

2X1 ~ *N*(2μ, 22σ2) => 2X1 ~ *N*(2μ, 4σ2)

and following the property of addition,

X1 + X2 ~ *N*(μ+ μ,, σ2 + σ2) ~ N(2μ, 2σ2)

And the difference between the two is given by

2X1 – (X1 + X2) ~ *N*(2μ- 2μ,, 2σ12 + 4σ22) ~ N(0, 6σ2)

The mean of 2X1 and X1 + X2 is same but the var(σ2) of 2X1 is 2 times more than the variance of X1 + X2.

The difference between the two says that the two given variables are identically and independently distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

Given: p(a<x<b) = 0.99, mean =100, and Standard Deviation = 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is 0.99.

From the above details, we must exclude area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values.

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.576) + 100= 48.5

b = (20\*2.576) + 100= 151.5

Two values symmetric about mean for the given standard normal distribution are 48.5 and 151.5.

Hence, the correct option is D. 48.5, 151.5.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

A): Range containing 95% probability for profit of company is.

(Rs. 99M, Rs. 1026M).

B): Rs. 170.1 million.

C): First division of the company has larger probability of making a loss.

Given that:

$1 = Rs. 45

Profit1 ~ N(5, 3^2)

Profit2 ~ N(7, 4^2)

Thus,

Company's profit:

P ~ N( 5+7, 3^2 + 4^2) = N(12, 5^2)

A):

95% of the probability lies between 1.96 standard deviations of the mean.

Thus range is:

= (12 - 1.96 \* 5, 12 + 1.96 \* 5)

= ($2.2M, $22.8M)

= (Rs. 99M, Rs. 1026M)

B): Fifth percentile is calculated as:

P(Z <= (p-12)/5) = 0.05

From p values of z score table, we get:

(p-12)/5 = -1.644

p = 12 - 8.22 = 3.78

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies.

Or 5th percentile of profit is Rs. 170.1 Million.

C): Loss is when profit < 0

Thus: p < 0

The first division of company, thus have larger probability of making a loss in a given year.